Topography optimization of a plate coupled with acoustic cavity

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This paper is dedicated to the memory of Professor Liviu Librescu who has touched
and enriched our lives as a great scholar, consummate professional, and most
importantly as a wonderful friend

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Finite element
Experimental validation

Abstract

Optimization of the topology of a plate coupled with an acoustic cavity is presented in an
attempt to minimize the fluid–structure interactions at different structural frequencies. A
mathematical model is developed to simulate such fluid–structure interactions based on
the theory of finite elements. The model is integrated with a topology optimization
approach which utilizes the moving asymptotes method. The obtained results demonstrate
the effectiveness of the proposed approach in simultaneously attenuating the structural
vibration and the sound pressure inside the acoustic domain at several structural frequen-
cies by proper redistribution of the plate material.

Experimental verification is carried out by manufacturing topology optimized plates and
monitoring their vibration and sound radiation into a rigid acoustic cavity. The measured
sound pressure and plate vibration are found to be in good agreement with the predictions
of the mathematical model.

The presented theoretical and experimental techniques present valuable tools in the
design of a wide variety of critical structures which must operate quietly when subjected
to fluid loading.

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1. Introduction

In recent years, extensive application of topology optimization to continuum structures has been reported, as it has been
recognized that using such an approach yields to improved static and dynamic characteristics. For these reasons, topology
optimization has found its ways in aeronautical, civil and mechanical engineering applications, and started to become a stan-
dard module of commercial finite element packages.

The literature on structural topology optimization is quite extensive and the research activities in this field have focused on a
wide variety of applications. The optimization problem was treated as a material distribution problem to minimize/maximize
certain objective functions. In other words, the structural material is redistributed to achieve the optimization goal bounded by
various constraints, among which is the volume fraction of the material. The efficiency of this method was clearly demonstrated
by solving the classical problem of minimizing the compliance of various structures (Bendsoe and Kikuchi, 1988; Bendsoe,
1989) and maximizing their fundamental buckling load (Bendsoe and Sigmund, 2003). Later on, structural dynamics started
become the focus of researchers working in topology optimization. Maximization of the structural dynamics properties such as
the eigenfrequencies and maximizing the gap between two consecutive eigenfrequencies was tackled by Bendsoe and Diaz
ical response of a structure for a given driving frequency or frequency range was studied by Jog (2002).

However, fewer studies have considered the application of topology optimization to fluid–structure interaction problems.
For example, Yoon et al. (2007) used a mixed finite element model to represent a fluid–structure coupled domain, where the
structure was placed inside an acoustic medium. Using this approach, the authors were able to formulate the problem without explicit boundary interface representation. The objective of the optimization scheme was to minimize the sound pressure inside the acoustic medium, when exciting the structure by fixed excitation frequency. Du and Olhoff (2007) have attempted to minimize the sound power radiated from a structure surface placed inside an acoustic cavity. However, because air was the acoustic medium, a feedback coupling between the acoustic medium and the structure was neglected. In addition, weak coupling was assumed and the acoustic pressure was ignored in the structural equations. Also, in their study the excitation frequency was maintained fixed at certain value regardless of the effect that the material redistribution has on the stiffness of the structure domain, and on the modal frequencies.

Hence, topology optimization of fluid–structure interaction problems, where true coupling is considered and the external excitation being locked to the modal frequencies has yet to considered. It is therefore the objective of the current paper to model a fluid–structure interaction problem, where a vibrating flexible plate is coupled to a closed acoustic cavity. A topology optimization approach using the method of moving asymptotes (MMA) (Svanberg, 1999; Bruyneel et al., 2002) will be developed to minimize the coupling between the flexible plate and the acoustic domain. Doing so, the energy transfer between the two domains will be minimized. All of this is considered at modal frequencies, since it is clear, that during the optimization phase the material distribution varies from iteration to the next, affecting consequently the natural frequencies of the fluid-loaded structure. In this case, the true attenuation of the structural vibration and the associated sound radiation can be attributed only to the effect of the topology optimization and not to the shift of the natural frequencies.

This paper is organized in six sections. In Section 1, a brief introduction has been presented. The mathematical model describing the fluid–structure interaction between the plate and the acoustic cavity is presented in Section 2. The formulation of the topology optimization problem is developed in Section 3. The theoretical and experimental performance characteristics of the topology optimized plate/cavity system are presented in Sections 4 and 5 respectively. A brief summary of the conclusions is given in Section 6.

2. Modeling of plate–cavity interaction

2.1. Overview

Consider the plate–cavity system shown schematically in Fig. 1. In this system, a rectangular flexible plate is coupled with an acoustic cavity that has five rigid walls. The plate is subjected to external excitation and a finite element model will be developed to predict the interaction between the plate vibration and the associated sound radiation into the acoustic cavity.
The finite element model consists of two different types of elements. The first one is 4-node quad elements with 3 degrees of freedom per node \((w, \theta_x, \theta_y)\) representing the transverse displacement of the node and rotations about the \(x\)-axis and the \(y\)-axis, respectively. The second type of elements is a cubic 8-node element for the acoustic domain with the acoustic pressure \(p\) as the sole degree of freedom per node. The element shapes are as presented in Figs. 2 and 3.

2.2. Individual models of the plate and cavity

2.2.1. The cavity

Considering a fluid volume \(V\), then the following identities can be defined:

\[
\text{Potential energy } (P.E.) = \frac{1}{2} \rho A c^2 \int_V \left( \text{div} u_A \right)^2 dV,
\]

\[\tag{1}\]

\[
\text{Kinetic energy } (K.E.) = \frac{1}{2} \rho A \int_V \dot{u}_A^2 dV.
\]

\[\tag{2}\]

The work done on the acoustic cavity by the plate element is given by

\[
(W_P) = \int_{\text{Boundary Area}} p w dA.
\]

\[\tag{3}\]

For the sake of simplifying the calculations, the velocity potential \(\phi\) (a scalar quantity) is used instead of the acoustic pressure \(p\) using the following identities:

\[
\text{Velocity potential } \phi = \int p dA
\]

\[
\text{Acoustic pressure } p = \int \phi \delta \rho u dA
\]

\[
\text{Acoustic pressure } p = \int \phi \delta \rho \dot{u}_A dA
\]

\[
\text{Acoustic pressure } p = \int \phi \delta \rho \dot{u}_A dA
\]

\[
\text{Acoustic pressure } p = \int \phi \delta \rho \dot{u}_A dA
\]
\[ \bullet \dot{p} = -\rho_A \frac{\partial \varphi}{\partial t}, \]
\[ \bullet \dot{u}_A = -\nabla \varphi = -\left[ \frac{\partial \varphi}{\partial x} i + \frac{\partial \varphi}{\partial y} j + \frac{\partial \varphi}{\partial z} k \right], \]
\[ \bullet \ddot{u}_A = \left( -\nabla \varphi \right)^2 = \left[ \left( \frac{\partial \varphi}{\partial x} \right)^2 + \left( \frac{\partial \varphi}{\partial y} \right)^2 + \left( \frac{\partial \varphi}{\partial z} \right)^2 \right]. \]
\[ \bullet \text{div} u_A = -\frac{1}{c^2} \frac{\partial \varphi}{\partial t}. \]

Hence, \( P.E. = \frac{1}{2} \rho_A \frac{c^2}{L} \int_V \dot{\varphi}^2 dV, \)
\[ \text{K.E.} = \frac{1}{2} \rho_A \int_V \left( \nabla \varphi \right)^2 dV, \]
and \( W_p = -\rho_A \int_{\text{Boundary}} \varphi^* w dA. \)

Hamilton’s principle, as given by Eq. (11), is used to extract the differential equation of motion of the acoustic fluid, as influenced by the coupling with the plate:

\[ \int_{t_1}^{t_2} \delta (P.E. - P.E. + W_p) dt = 0. \]

Let \( \varphi = N_v \varphi \) where \( N_v \) denotes an appropriate shape function and \( \varphi \) represents the nodal velocity potential vector of the element. Similarly, let \( w = N_w \{ \delta \} \) where \( N_w \) is an appropriate shape function and \( \{ \delta \} \) denotes the nodal deflection vector of the plate. Then

The variation of the potential energy (\( \delta \ P.E. \))

\[ \int_{t_1}^{t_2} \delta (P.E.) dt = \frac{\rho_A}{c^2} \int_{t_1}^{t_2} \int_V \{ \delta \varphi \}^T \left( \frac{\partial (N_v)^T (N_v)}{\partial t} \right) \{ \varphi \} dV dt. \]

Integrating Eq. (12) by parts to eliminate \( \{ \delta \varphi \} \) yields

\[ \int_{t_1}^{t_2} \delta (P.E.) dt = -\rho_A \int_{t_1}^{t_2} \int_V \{ \delta \varphi \}^T \left( \nabla N_v \right)^T \{ \nabla \varphi \} \{ \varphi \} dV dt. \]

Similarly, the variation of the kinetic energy (\( \delta \ K.E. \)):

\[ \int_{t_1}^{t_2} \delta (K.E.) dt = \rho_A \int_{t_1}^{t_2} \int_V \{ \delta \varphi \}^T \left( \frac{\partial (N_v)^T (N_v)}{\partial t} \right) \{ \varphi \} dV dt, \]
and the variation of the work done by the plate (\( \delta W_p \)) can be determined as follows:

As \( W_p = -\rho_A \int_A \left( \frac{\partial (N_v)^T (N_v)}{\partial t} \right) \{ \varphi \} dA \)

then \( \int_{t_1}^{t_2} \delta (W_p) dt = -\rho_A \int_{t_1}^{t_2} \int_A \{ \delta \varphi \}^T \left( \frac{\partial (N_v)^T (N_v)}{\partial t} \right) \{ \varphi \} dA dt. \)

or \( \int_{t_1}^{t_2} \delta (W_p) dt = -\rho_A \int_{t_1}^{t_2} \int_A \{ \delta \varphi \}^T \left( \nabla N_v \right)^T \{ \nabla \varphi \} \{ \varphi \} dA dt. \)

Finally, summing up the terms of \( \{ \delta \varphi \} \) inside the time integral and equating them to zero results in the required equation of motion of the acoustic element:

\[ \frac{\rho_A}{c^2} \int_V (N_v)^T \{ \varphi \} dV + \rho_A \int_V (\nabla N_v)^T \{ \nabla \varphi \} dV + \rho_A \int_A \{ \varphi \} dA \}

Differentiating with respect to time and utilizing the following identities:

\[ \dot{\varphi} = -\frac{\varphi}{m}, \quad \ddot{\varphi} = -\frac{\ddot{\varphi}}{m}, \quad \\text{and} \quad \dddot{\varphi} = -\frac{\dddot{\varphi}}{m}. \]

gives \( \frac{1}{c^2} \int_V (N_v)^T (N_v) dV \{ \dot{p} \} + \int_V (\nabla N_v)^T (\nabla \varphi) dV \{ \ddot{\varphi} \} = -\rho_A \int_A (N_v)^T (N_v) dA \{ \dddot{\varphi} \}. \)

where \( \{ p \} \) is the nodal pressure vector, which could also be written as

\[ \{ M_A \} \{ \dot{p} \} + \{ K_A \} \{ p \} = -\{ \Omega \} \{ \dddot{\varphi} \}. \]
where

- \( \{ \delta \} \) = the degrees of freedom of the structure element,
- \( [M_A] = \frac{1}{2} \int_V (N_\alpha)^T (N_\alpha) \, dV \) = the mass matrix of the acoustic medium,
- \( [K_A] = \int_V (\nabla N_\alpha)^T (\nabla N_\alpha) \, dV \) = the stiffness matrix of the acoustic medium,
- \( [\Omega] = \rho_A f h [(N_\alpha)^T (N_\alpha)] \, dA \) = the fluid–structure coupling matrix.

2.2.2. The plate

The plate finite element model is based on the first order shear deformation theory, which is efficiently used for relatively thick as well as thin plates. In this model, it is assumed that planes normal to the mid-surface of the plate in the un-deformed state remain plane but not necessarily normal to the mid-surface in the deformed state. Hence the rotation degrees of freedom (\( \theta_x, \theta_y \)) are considered as independent degrees of freedom and not derivatives of the mid-surface out of plane displacements.

Then, the equation of motion of the plate is given as

\[
[M_A] \{ \delta \} + [K_A] \{ \delta \} = F_s,
\]

where \( F_s \) represent the forces exerted by the acoustic fluid on the plate elements.

The forcing function “\( F_s \)” can be calculated from the work done by the acoustic field on the structure as follows:

\[
W_p = \int_{\text{Boundary/area}} \omega \rho dA
\]

\[
W_p = \int_A [\delta]^T (N_w)^T (N_w) (\mathbf{p}) \, dA
\]

Hence, \( \int_{t_1}^{t_2} \delta (W_p) dt = \int_{t_1}^{t_2} \int_A [\delta \delta]^T (N_w)^T (N_w) (\mathbf{p}) \, dA \, dt \).

Since the work = force \times \text{displacement}, then the forcing term on the plate can be calculated as

\[
F_s = \frac{[\Omega][\mathbf{p}]}{\rho_A}
\]

Thus, the complete differential equation of the plate is given as

\[
[M_A] \{ \delta \} + [K_A] \{ \delta \} = \frac{[\Omega][\mathbf{p}]}{\rho_A}.
\]

2.3. Coupling the acoustic cavity with the plate structure

The equation of motion of the coupled system is given in the following matrix form

\[
\begin{pmatrix}
[M_A] & 0 \\
[\Omega]^T & [M_A]
\end{pmatrix}
\begin{pmatrix}
\delta \\
\mathbf{p}
\end{pmatrix}
+ \begin{pmatrix}
[K_A] & -[\Omega]/\rho_A \\
0 & [K_A]
\end{pmatrix}
\begin{pmatrix}
\delta \\
\mathbf{p}
\end{pmatrix}
= \begin{pmatrix}
f_s \\
0
\end{pmatrix},
\]

where “\( f_s \)” is the externally applied force.

From (26) for harmonic excitation at angular frequency \( \omega \)

\[
(K_A - M_A \omega^2) \frac{\Omega}{\rho_A} \mathbf{p} = f_s
\]

or, \( \mathbf{p} = (K_A - M_A \omega^2)^{-1} \omega^2 \Omega^T \delta \).

Substituting for \( \mathbf{p} \) from (28) into (27) results in

\[
(K_A - M_A \omega^2) \frac{\Omega}{\rho_A} (K_A - M_A \omega^2)^{-1} \omega^2 \Omega^T \delta = f_s.
\]

It is important here to note that a boundary layer is inserted at the interface between the plate and the cavity as shown in Fig. 4. This boundary layer accommodates the changes between the plate nodal DOF (w, \( \theta_x, \theta_y \)) and the cavity nodal DOF (p). In this layer, the out of plane displacement of the plate causes the adjacent fluid to move with the same velocity in a direction normal to the plate surface, thus resulting in a pressure variation inside that layer which is then transmitted to neighboring fluid elements. In return, the plate is loaded with the fluid mass adjacent to resulting in variation of the stiffness and mass of the fluid loaded plate. Therefore the coupling matrix “\( \Omega \)” described in Eqs. (20) and (25), is developed to account for this mutual interaction.

3. Formulation of the optimization of the fluid–structure interaction problem

3.1. Problem formulation

The topology optimization problem can be formulated as follows:

$$\min_{q} J = \left( \delta^T C_w \delta \right)$$

defined at the boundary area

$$\begin{bmatrix} M_{st} & 0 \\ \Omega^T & M_{A} \end{bmatrix} \begin{bmatrix} \delta \\ \bar{p} \end{bmatrix} + \begin{bmatrix} K_{st} & -[\Omega]/\rho_s \\ 0 & K_{A} \end{bmatrix} \begin{bmatrix} \delta \\ \bar{p} \end{bmatrix} = \begin{bmatrix} f_s \\ 0 \end{bmatrix}.$$ 

$$\sum_{e=1}^{N_e} h^T A^e \leq v_f \times h_{max} \times \sum_{e=1}^{N_e} A^e$$

$$0 < h_{min} < h^e < h_{max}$$

Note that $C_w \delta = \bar{w}$ where $C_w$ is a matrix that extracts the transverse deflection $\bar{w}$ from the nodal deflection vector $\delta$. Also, the objective function can be explained literally as to minimize the coupling between the structure and the fluid domains. By doing this, minimum work or energy is transmitted between the structure and the fluid domains, resulting in minimization of the sound intensity and sound pressure levels, due to structure resonances inside the acoustic cavity. It is worth mentioning, that due to the formulated objective function, the cavity modes will not be affected as a result of the topology optimization of the flexible plate.

3.2. Sensitivity analysis

The sensitivity of the objective function “J” which defines the fluid–structure coupling, with respect to the optimization variable (plate element density “$\rho$”) is given as follows:

$$\frac{dJ}{d\rho} = \delta^T C_w \frac{dp}{d\rho} + \bar{p}^T C_w \frac{d\delta}{d\rho}.$$

To calculate $\frac{d\bar{p}}{d\rho}$ we differentiate Eq. (29) w.r.t. $\rho$. This yields

$$\frac{d}{d\rho} \left[ \left( K_s - M_A \omega^2 \right)^{-1} \frac{\partial}{\partial \rho} \left( K_s - M_A \omega^2 \right)^{-1} \omega^2 \left( \Omega^T \right) \right] \delta \frac{d\bar{p}}{d\rho} = 0.$$

Define $K_{s,D} = (K_s - M_A \omega^2)$, $K_{A,D} = (K_A - M_A \omega^2)$, $A = \frac{\partial}{\partial \rho} \left( K_s - M_A \omega^2 \right)^{-1} \omega^2 \left( \Omega^T \right)^T$

$$\frac{dK_{s,D} \delta + A \frac{d\delta}{d\rho}}{d\rho} = 0, \quad B = \left[ K_{s,D} - A \right], C = \frac{dK_{A,D}}{d\rho}.$$

Hence,$\frac{d\bar{p}}{d\rho} = -B^{-1} C \delta$.

To calculate $\frac{dp}{d\rho}$ we differentiate Eq. (28) with respect to $\rho$ gives

$$\omega^2 \left( \Omega^T \right)^T \frac{d\delta}{d\rho} = (K_s - M_A \omega^2) \frac{dp}{d\rho}.$$

Define $D = \omega^2 \left( \Omega^T \right)^T$, then $\frac{dp}{d\rho} = K_{A,D}^{-1} D \frac{d\delta}{d\rho}$.
During the topology optimization calculation, the effect of the density is substituted for by the effect of the thickness of the plate elements.

4. Numerical results

4.1. Model parameters

A finite element model for a closed acoustic cavity coupled with a flexible aluminum plate was developed. The characteristics of the coupled fluid–structure domains are as given in Table 1 and as shown in Fig. 1.

During the topology optimization calculation, the effect of the density is substituted for by the effect of the thickness of the plate elements.

4.2. Excitation frequency and topology optimization targeting the first odd mode

The shape of material distribution and relative plate displacement field progresses as shown in Fig. 5. While the actual objective function was to minimize the fluid–structure acoustic coupling, it was more realistic to monitor the effect of the optimization process on the sound intensity measured inside the acoustic cavity at the frequency of the first odd structural mode. Fig. 6 illustrates the solution convergence for the first mode optimization and its effect on the attenuation of the sound intensity.

4.3. Excitation frequency and topology optimization targeting the second odd mode

The shape of material distribution and relative plate displacement field progresses as shown in Fig. 9.

The frequency response for the plate displacement and the sound pressure were again monitored at the same locations as in the first case and are plotted in Figs. 11 and 12.

Table 1
Coupled structure–fluid coupled domains parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavity dimensions</td>
<td>12&quot; × 12&quot; × 30&quot;</td>
</tr>
<tr>
<td>Fluid domain</td>
<td>Air at 25 °C and 1 atm</td>
</tr>
<tr>
<td>Flexible plate dimensions</td>
<td>12&quot; × 12&quot;</td>
</tr>
<tr>
<td>Flexible plate thickness (h_{max})</td>
<td>1/16&quot;</td>
</tr>
<tr>
<td>Flexible plate material</td>
<td>Aluminum</td>
</tr>
<tr>
<td>Volume fraction, v_f</td>
<td>0.5</td>
</tr>
<tr>
<td>h_{min}</td>
<td>0.25 × h_{max}</td>
</tr>
</tbody>
</table>
It can be seen from the aforementioned figures that by exciting the aluminum plate at the first structural mode frequency, the plate displacement is considerably reduced as well as the sound pressure inside the acoustic cavity. The sound pressure attenuation at the first mode has measured above 3.5 dB. Considerable attenuation was also monitored at other structural modes. When targeting the second structural odd mode on the other hand, the sound pressure attenuation measured at that mode has recorded a value above 12 dB, although the optimization didn’t significantly affect the attenuation at other structural modes.
5. Experimental verification

To verify the obtained results experimentally, a set of three different Aluminum plate configurations were prepared. The first plate has surface dimensions of 12 in. × 12 in. and uniform thickness of 1/32 in. The second and third plates were manufactured from 1/64 in. Aluminum sheets to approximate the topology optimization results when targeting the first and second structural odd modes respectively, while maintaining the volume equal to the first plain case as shown in Figs. 13 and 14.

In Figs. 13 and 14, the plain uncovered metal has a thickness of 1/64 in., the while-covered parts have a thickness of 1/32 in., the red-covered parts have a thickness of 3/64 in. and finally the black-covered plate parts have a thickness of 1/16 in. Note that the plain plate weighs 254 g, topology optimized plate targeting the first mode is 252 g, and topology optimized plate targeting the fifth mode 256 g.

A 12” × 12” × 30” closed acoustic cavity was prepared. The cavity has only one surface coupled to the flexible aluminum plate as shown in Fig. 15. Each of the three different plates was mounted and the plate acceleration as well as the sound pressure level inside the cavity was measured. The plate was mechanically excited with a speaker that is mounted in a position to cause the excitation to be symmetric.

The speaker was excited with a function generator that sweeps a frequency range between 40 Hz and 1 kHz with a resolution of 0.5 Hz. The acceleration was measured at the midpoint of the plate and the sound pressure was measured at a point corresponding to the midpoint of the plate, only 3 in. away.

Frequency response for the plate acceleration as well as the sound pressure inside the acoustic cavity are shown in Figs. 16 and 17 for the first case, where the optimization is tailored to the first structural odd mode.

The displayed results emphasize the effectiveness of the topology optimization in attenuating both the structural vibration of the plate and the sound pressure level inside the acoustic cavity. Furthermore, the obtained results agree closely with the theoretical predictions displayed in Figs. 7 and 8.

Figs. 18 and 19 show the corresponding results when the topology optimization is tailored to the second odd mode.

The obtained results demonstrate also the effectiveness of the topology optimization in attenuating both the structural vibration of the plate and the sound pressure level inside the acoustic cavity. Furthermore, the obtained results agree closely with the theoretical predictions displayed in Figs. 11 and 12.
In addition to frequency response measurement for the plate vibration acceleration and sound pressure inside the acoustic cavity, the plate displacement field was measured using a laser vibrometer. The setup is shown in Fig. 20. The three plate sets were excited at the first structural odd mode using a function generator and the excitation speaker. The laser vibrometer is used to measure the displacement field of the different plates, while locked at the same excitation frequency as shown in Figs. 21 and 22.

6. Conclusions

A topology optimization approach was developed for fluid–structure interaction between a flexible plate coupled with a rigid acoustic cavity. The objective of the optimization was to redistribute the material of the flexible plate in order to min-
imize the fluid–structure coupling. A finite element model was developed to simulate the fluid–structure interactions and was integrated with the topology optimization approach. The model was used to develop the sensitivity analysis necessary for the operation of the topology optimization algorithm. The excitation acting on the plate was locked at the first or second structural odd modes to ensure the effectiveness of the optimization in reducing the sound pressure at the modal frequencies. The analytical model showed considerable attenuation for the first structural odd mode as well as consecutive modes, when the optimization was targeting that mode specifically. On the other hand excellent attenuation was obtained for the second structural odd mode, when targeting the optimization scheme towards that specific mode.

Fig. 11. Displacements of plain plate and topology optimized plate targeting second odd mode.

Fig. 12. Sound pressure of plain plate and topology optimized plate targeting second odd mode.

Fig. 13. Manufactured plate approximating the optimization results when targeting the first odd mode.

Fig. 14. Manufactured plate approximating the optimization results when targeting the second odd mode.

Fig. 15. Experimental setup.

Fig. 16. Experimental sound pressure of plain plate and topology optimized plate targeting first odd mode.
Experimental verification was carried out by manufacturing a set of topology optimized plates that approximate the results obtained from the analytical model. The plates were coupled to an acoustic cavity. Plate vibration acceleration and sound pressure inside the acoustic cavity were measured and compared with the plain-plate case. Considerable attenuation in both the plate vibration acceleration and sound pressure inside the acoustic cavity were recorded. A good match with the analytical model was observed. The displacement fields of the plate were also measured using a laser vibrometer, and good agreement was observed with the analytical case.

The presented topology optimization approach can be an invaluable tool in the design of a wide variety of critical structures which must operate quietly when subjected to fluid loading. Note that the utility of such a design tool is enhanced through the use of the first order shear deformation theory which makes the analysis equally applicable to thin and thick plates.

**Fig. 17.** Experimental accelerations of plain plate and topology optimized plate targeting first odd mode.

**Fig. 18.** Experimental sound pressure of plain plate and topology optimized plate targeting second odd mode.

**Fig. 19.** Experimental accelerations of plain plate and topology optimized plate targeting second odd mode.
Fig. 20. Laser vibrometer experimental setup.

Fig. 21. Displacement field for the plain plate excited at the first structural odd mode.

Fig. 22. Displacement field for the plate optimized for the first structural odd mode.
plate structures. Therefore, a natural extension of the present work is to theoretically predict and experimentally validate the performance of topology optimized thick plates coupled with acoustic cavities.

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References